Non-hermitian Hamiltonians

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Chain model with one type of sites

$$H = -\sum_l \left(t^+ a^\dagger_{l+1} a_l + t^- a^\dagger_{l-1} a_l \right)$$

$$H^\dagger = -\sum_l \left((t^+)^* a^\dagger_{l+1} a_l + (t^-)^* a^\dagger_{l-1} a_l \right)$$

Parity transformation and the time reversal:

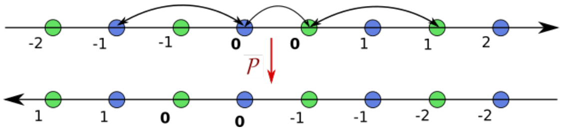
$$\mathcal{P} a_l \mathcal{P} = a_{-l}, \qquad \mathcal{P} t^\pm \mathcal{P} = t^\pm$$

$$\mathcal{T} a_l \mathcal{T} = a_{-l}, \qquad \mathcal{T} t^\pm \mathcal{T} = (t^\pm)^*$$

$$H^{\mathcal{PT}} = -\sum_l \left((t^+)^* a^\dagger_{l+1} a_l + (t^-)^* a^\dagger_{l-1} a_l \right)$$

Hermiticity: $t^- = (t^+)^*$
$\mathcal{PT}$ invariance: $t^- = (t^+)^*$

Chain model with two sublattices



$$H = -\sum_l [t^+_{AB} b^\dagger_l a_l + t^-_{AB} b^\dagger_{l-1} a_l + t^+_{BA} a^\dagger_{l+1} b_l + t^-_{BA} a^\dagger_l b_l$$

$$+ t^+_{AA} a^\dagger_l a_{l+1} + t^-_{AA} a^\dagger_l a_{l-1} + t^+_{BB} b^\dagger_l b_{l+1} + t^-_{BB} b^\dagger_{l-1} b_l$$

Hermiticity:

$$\begin{cases} (t^+_{AB})^* = t^-_{BA} \\ (t^+_{BA})^* = t^-_{AB} \\ (t^+_{AA})^* = t^-_{AA} \\ (t^+_{BB})^* = t^-_{BB} \end{cases}$$

$\mathcal{PT}$ invariance:

$$\begin{cases} (t^+_{AB})^* = t^-_{AB} \\ (t^+_{BA})^* = t^-_{BA} \\ (t^+_{AA})^* = t^-_{AA} \\ (t^+_{BB})^* = t^-_{BB} \end{cases}$$

The model

Choose the following set of parameters:

$$t^+_{AA} = -t^-_{AA} = it_A, \qquad t^+_{BB} = -t^-_{BB} = it_B,$$
$$t^+_{AB} = -t^-_{AB} = ig_1, \qquad t^+_{BA} = -t^-_{BA} = ig_2,$$

$$H = -i\sum_l (g_1(b^\dagger_l - b^\dagger_{l-1})a_l + g_2(a^\dagger_{l+1} - a^\dagger_l)b_l$$
$$+t_A(a^\dagger_{l+1} - a^\dagger_{l-1})a_l + t_B(b^\dagger_{l+1} - b^\dagger_{l-1})b_l)$$

The eigensystem is determined by matrix equation:

$$\mathcal{H}_p\Psi_p = \epsilon(p)\Psi_p,$$

where

$$\mathcal{H}_p = -2\begin{pmatrix} t_A \sin p & g_2 e^{-\frac{ip}{2}} \sin\frac{p}{2} \\ g_1 e^{\frac{ip}{2}} \sin\frac{p}{2} & t_B \sin p \end{pmatrix}$$

Eigenenergies:

$$\epsilon_\pm(p) = -(t_A + t_B)\sin p \pm sign(p)\sqrt{(t_A - t_B)^2 \sin^2 p + 4g_1 g_2 \sin^2\frac{p}{2}}$$

Weyl modes in a single closed Dirac cone

Expanding Hamiltonian around the origin:

$$\mathcal{H}_p^{(0)} = -2 \begin{pmatrix} 2t_A & g_2 \\ g1 & 2t_B \end{pmatrix} p$$

Diagonalize hamiltonian

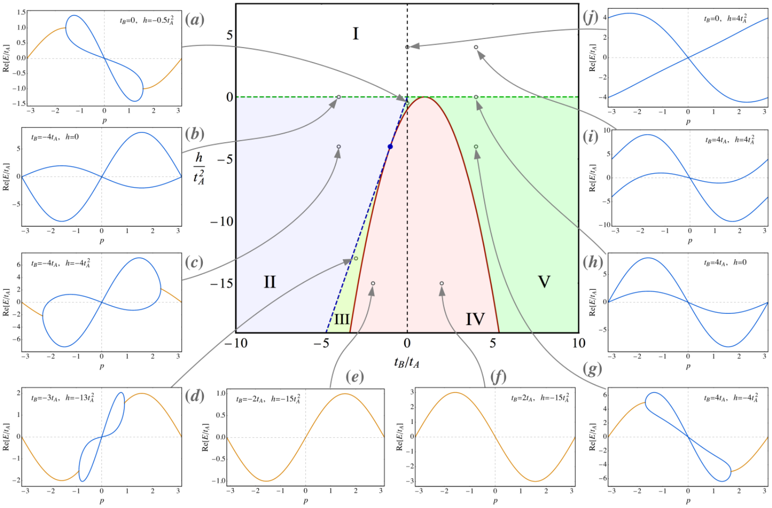
$$\mathcal{H}_p^{(0)} = U^\dagger \mathcal{H}_p^{diag} U$$

$$\mathcal{H}_p^{diag} = \begin{pmatrix} \upsilon_+ & 0 \\ 0 & \upsilon_- \end{pmatrix}$$

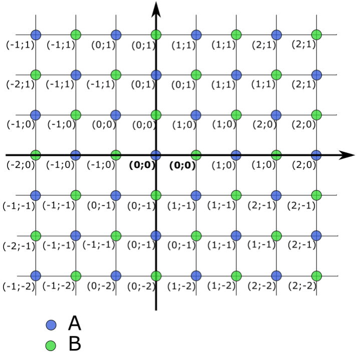
Then solutions of eigenvalue equation satisfy two-component Weyl equations:

$$\left(\gamma^0 \frac{1}{\upsilon_\pm} \frac{\partial}{\partial t} + \gamma^1 \frac{\partial}{\partial x} \right) \psi_\pm(t, x) = 0$$

Energy spectrum



Two-dimensional lattice



$$
\begin{aligned}
H = -2i \sum_{x,y} \big[& g_1 (b^\dagger_{x,y} + b'^\dagger_{x,y} - b^\dagger_{x-1,y} - b'^\dagger_{x,y-1}) a_{x,y} \\
& + g_2 (a^\dagger_{x+1,y} + a'^\dagger_{x+1,y} - a^\dagger_{x,y} - a'^\dagger_{x+1,y-1}) b_{x,y} \\
& + t_A (a^\dagger_{x+1,y} + a^\dagger_{x,y+1} - a^\dagger_{x-1,y} - a^\dagger_{x,y-1}) a_{x,y} \\
& + t_B (b^\dagger_{x+1,y} + b^\dagger_{x,y+1} - b^\dagger_{x-1,y} - b^\dagger_{x,y-1}) b_{x,y} \big]
\end{aligned}
$$

$$
\mathcal{H}_p = -4 \begin{pmatrix} t_A(\sin p_x + \sin p_y) & g_2\left(e^{-i\frac{p_x}{2}} \sin\frac{p_x}{2} + e^{-i\frac{p_y}{2}} \sin\frac{p_y}{2}\right) \\ g_1\left(e^{i\frac{p_x}{2}} \sin\frac{p_x}{2} + e^{i\frac{p_y}{2}} \sin\frac{p_y}{2}\right) & t_B(\sin p_x + \sin p_y) \end{pmatrix}
$$

Eigenenergies:

$$
\begin{aligned}
\epsilon = & -2(t_A + t_B)(\sin p_x + \sin p_y) \\
& \pm 2\sqrt{(t_A - t_B)^2 (\sin p_x + \sin p_y) + 4h\left(\sin^2\frac{p_x}{2} + \sin^2\frac{p_y}{2} + 2\sin\frac{p_x}{2}\sin\frac{p_y}{2}\cos\frac{p_x - p_y}{2}\right)}
\end{aligned}
$$

Energy spectrum