# Precursors of QCD Phase Transitions in Hot and Dense Matter

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# (Classical) QCD Lagrangian

$$\mathcal{L}^{cl} = \bar{q}(i\gamma^\mu D_\mu - m)q - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a$$

$$D_\mu = \partial_\mu - igt^a A^a_\mu \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu$$

$$q = {}^t(u, d, s, c, b, t) \qquad m = diag(m_u, m_d, m_{s,} ...)$$

**Quantum theory:**

Gauge fixing+ Fadeev-Popov ghost fields
Regularization with some scale $\mu$

Independence of physical values of observables of $\mu$

→ Renormalization group equation，which in turn describes the scale dependence of observables.

# Current divergences and Quantum Anomalies

From Noether's theorem:

$$\partial_\mu(\bar{q}\gamma^\mu\lambda^a q) = i\sum_{i,j}^{N_f}\bar{q}_i(m_i - m_j)\lambda^a q_j \quad (a = 0 \sim N_f^2 - 1)$$

$i, j =$u,d,s, …

$$\partial_\mu(\bar{q}\gamma^\mu\gamma_5\lambda^a q) = i\sum_{i,j}^{N_f}\bar{q}_i(m_i + m_j)\gamma_5\lambda^a q_j \quad (a = 1 \sim N_f^2 - 1)$$

$$\partial_\mu(\bar{q}\gamma^\mu\gamma_5 q) = i\sum_{i}^{N_f}\bar{q}_i 2m_i\gamma_5 q_i + 2N_f\frac{g^2}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$$

$\left(\tilde{F}^{\lambda\rho}_a = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F^a_{\mu\nu}\right)$

Chiral Anomaly

Quantum effects!

Dilatation

$$\partial_\mu D^\mu = \Theta^\mu_\mu = (1+\gamma_m)\sum_{i}^{N_f}\bar{q}_i m_i q_i + \frac{\beta}{2g}F^a_{\mu\nu}F^{\mu\nu}_a$$

Dilatation(scale) Anomaly

$\Theta_{\mu\nu}$; energy-momentum tensor of QCD

Some symmetries existing in the classical level are broken explicitly in the quantum level. Quantum Anomaly

# The notion of Spontaneous Symmetry Breaking

$Q^a$ the generators of a continuous transformation

$\partial^\mu j^a_\mu = 0$ ; $j^a_\mu(x)$ Noether current $Q^a = \int d\mathbf{x}\, j^a_0(x)$

eg. Chiral transformation for $SU_L(2) \otimes SU_R(2)$

$Q^a_5 = \int d\mathbf{x}\, \bar{q}\gamma^0\gamma_5\tau^a / 2q$ Notice; $[iQ^a_5, \bar{q}(x) i\gamma_5\tau^b q(x)] = -\delta^{ab}\bar{q}(x)q(x)$

The two modes of symmetry realization in the vacuum $|0\rangle$ :

a. Wigner mode $Q^a|0\rangle = 0 \quad \forall a$

b. Nambu-Goldstone mode $Q^a|0\rangle \neq 0 \quad \exists a$

The symmetry is spontaneously broken.

Now, $\langle 0|\bar{q}q|0\rangle = \langle 0|[Q^a_5, \bar{q}\gamma_5\tau^a q]|0\rangle$

$\langle 0|\bar{q}q|0\rangle \neq 0 \Rightarrow Q^a_5|0\rangle \neq 0$

Chiral symmetry is spontaneously broken!

# $U_A(1)$ Problem

# of the generators

G= $U_L(3) \otimes U_R(3)$ 2x(8+1)=18

↓

H= $U_V(1) \otimes SU_f(3)$ 1+8=9

# of NG-bosons= dim $G$ - dim $H = 18 - 9 = 9$ (?)

↑

Nambu-Goldstone Theorem

★ Number of the lightest pseudo-scalar mesons

$\pi^{\pm}, \pi^0$(140) $K^{\pm}, K^0, \overline{K^0}$(500) $\eta$(550) << $\eta'$ (958)

3 + 4 + 1 $= 8 \neq 9$ !

Why is $\eta'$ so massive ?

------ $U_A(1)$ Problem

c.f. Without the anomaly,

$m_{\eta_0} \leq \sqrt{3} m_\pi$

-- S.Weinberg('75) --

$\exists\ U_A(1)$ Anomaly $\partial_\mu(\bar{q}\gamma_\mu\gamma_5 q) = 2N_f \frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$ Operator Equation!

even in the chiral limit! $\neq 0$ + Instantons

# What is the matter?

According to modern QFT, the matter is an excited state of quantum fields.

The ground state is the vacuum.

Def. $a_\alpha |0\rangle = 0 \longleftrightarrow |0\rangle$ ; the vacuum

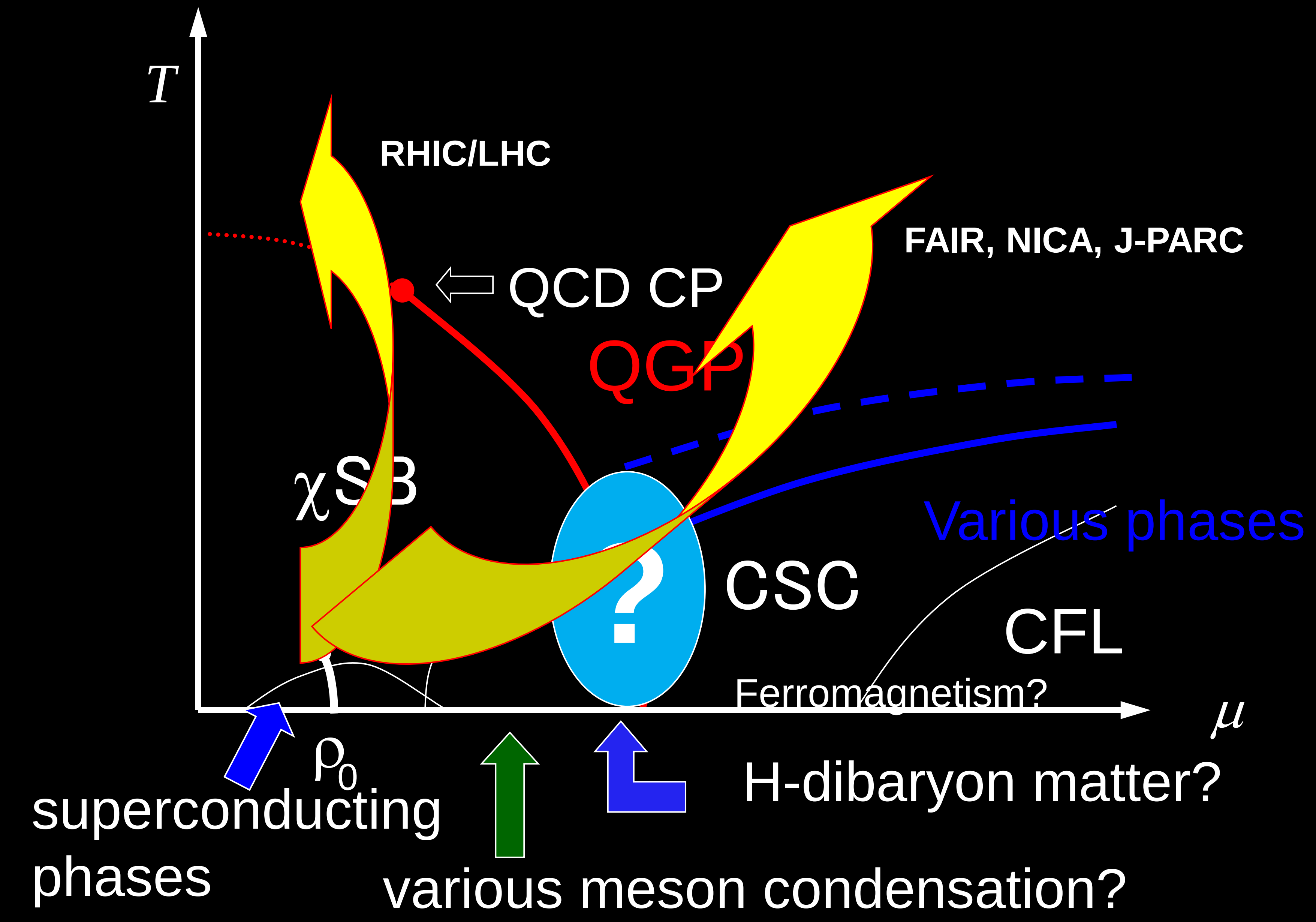
The vacuum is defined through the annihilation operators of the Matter.

The matter and the vacuum are inter-determined.
The modern theory of the matter is automatically the theory of the vacuum.
Determining what the matter is equivalent to determine the vacuum.

Eg. Superconductivity in metal

Quasi-electrons get a gap in dispersion rel. around Fermi surface.

# A conjectured QCD phase diagram



The absolute value to the quark condensate
is expected to decrease in average
in hot and/or dense matter.

# Finite T

$$\delta\langle\langle\bar{u}u\rangle\rangle \simeq \frac{\partial F_{\pi-\mathrm{gas}}}{\partial m_u} = 3\sum_{\boldsymbol{k}} \frac{\partial E_\pi(k)}{\partial m_u} n_\pi(k)$$

$$\frac{\partial E_\pi(k)}{\partial m_u} = \frac{m_\pi}{E_\pi(k)} \frac{\partial m_\pi}{\partial m_u} = \langle \pi(k)|\bar{u}u|\pi(k)\rangle$$

$$\langle\pi(0)|\bar{u}u|\pi(0)\rangle \simeq \langle\pi(0)|\bar{d}d|\pi(0)\rangle \simeq \frac{\partial m_\pi}{\partial \hat{m}} \simeq \frac{m_\pi}{m_u+m_d} = 7 \sim 10 > 0,$$

q-bar q probes either the vacuume or pions that are thermallly excite.:The llatter gives a lositive number, and thus the absolute value of the averaged condensate decreases.

# Finite density

$$\langle\Psi|\bar{q}q|\Psi\rangle = \frac{\partial\langle\Psi|\mathcal{H}_{\mathrm{QCD}}|\Psi\rangle}{\partial m_q}$$

$$\langle \mathrm{nm}|\mathcal{H}_{\mathrm{QCD}}|\mathrm{nm}\rangle = E_{\mathrm{vac}} + n_B[M_N + E_{\mathrm{b.}}],$$

$$\frac{\partial M_N}{\partial m_q} = \langle N| : \bar{q}q : |N\rangle$$

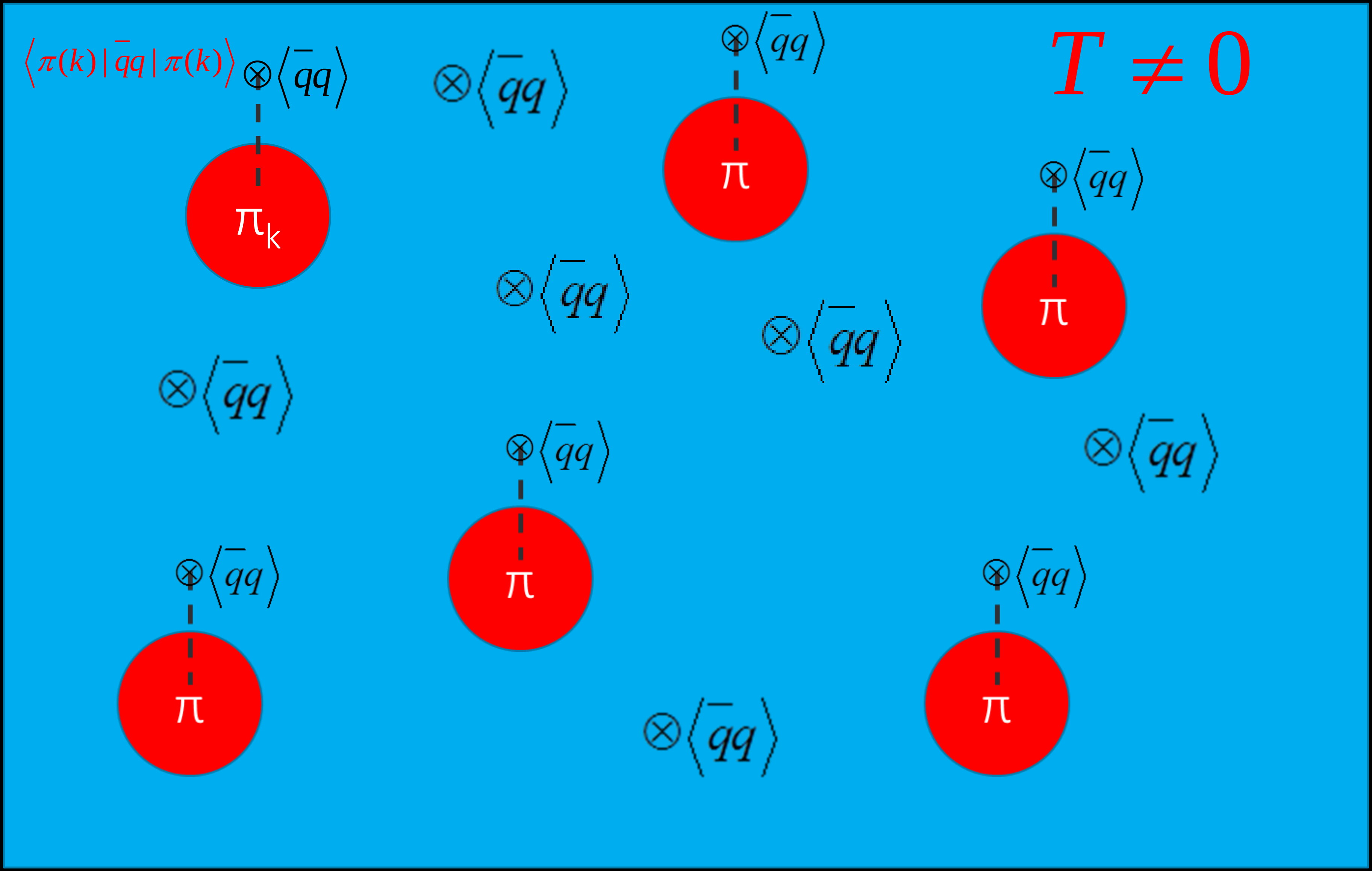
$$\Sigma_{\pi N} = \hat{m}\langle N| : \bar{u}u + \bar{d}d : |N\rangle$$

$$\frac{f_\pi^* {m_\pi^*}^2}{f_\pi {m_\pi}^2} = \frac{\langle \mathrm{nm}|\bar{u}u+\bar{d}d|\mathrm{nm}\rangle}{\langle\bar{u}u+\bar{d}d\rangle_0} \simeq 1 - \frac{n_B}{f_\pi^2 m_\pi^2}\Sigma_{\pi N}$$

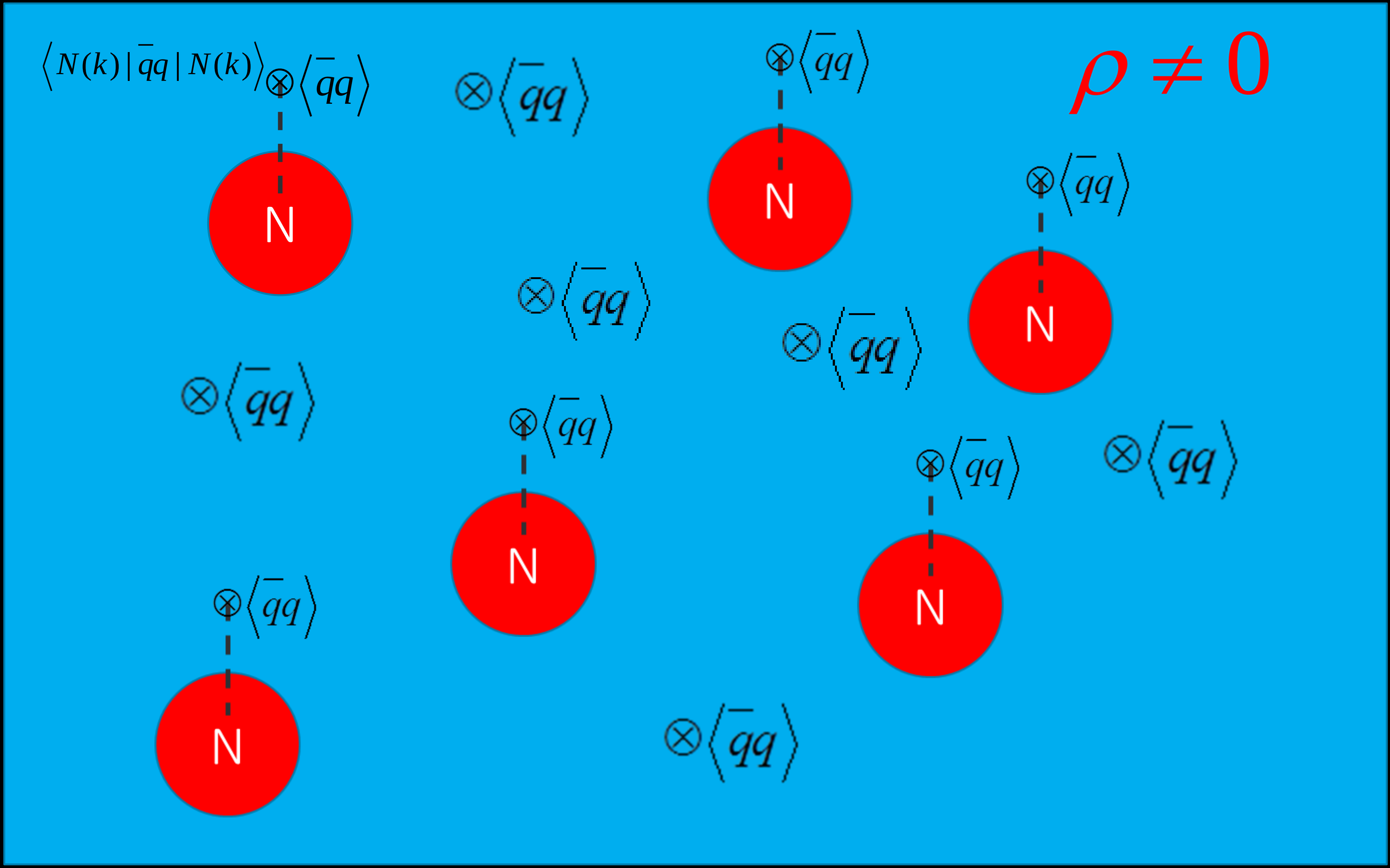
In the n uclear medium, nucleons play the same role as the pions do in hot matter.

$$\langle\bar{s}s\rangle_P = .53,$$

The $\bar{q}q$ operator probes thermaly excited pions with the B-E probability $n_\pi(k)$ as well as the vacuum at $T \neq 0$



The q-qbar operator probes nucleons with the F-D distribution function $f_N(k)$ as well as the vacuum at $\rho \neq 0$



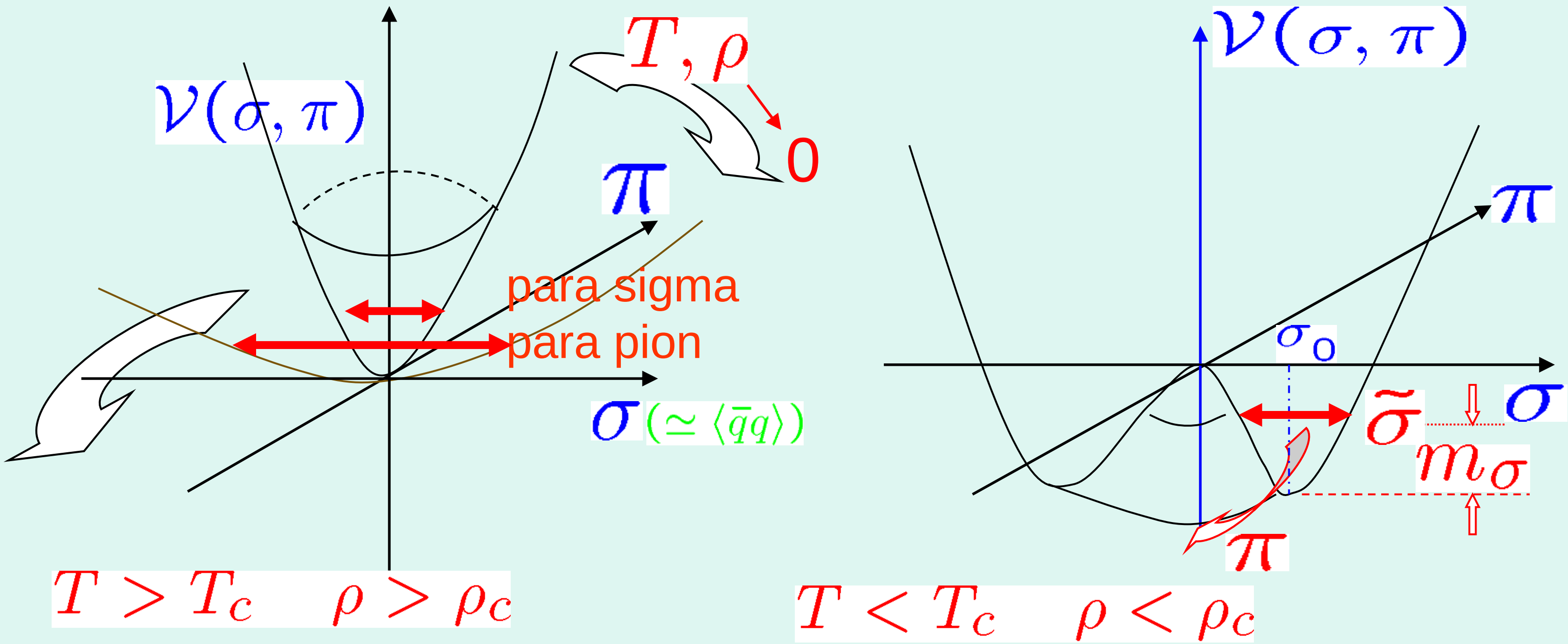
# Phenomena expected when chiral symm. Is (partially) restored.

Chiral restoration implies that correlators in the positive/negative parity get degenerate.

$$\langle S(x)S(y)\rangle \to \langle P^a(x)P^a(y)\rangle, \qquad \langle A^a_\mu(x)A^b_\nu(y)\rangle \to \langle V^a_\mu(x)V^b_\nu(y)\rangle$$

Scalar-Pseudoscalar Axial vector-Vector

# The sigma meson as a Higgs particle of chiral symmetry breaking in QCD




The low mass sigma in vacuum is now established:

pi-pi scattering; Colangero, Gasser, Leutwyler('06) and many others

Full lattice QCD ; SCALAR collaboration ('03)

**q-qbar, tetra quark, glue balls, or their mixed st's?**

M.Wakayama et al（SCALAR Collab.), PRD91(2015)

c.f. The sigma as the Higgs particle in QCD $\sigma = \sigma_0 + \tilde{\sigma}$ ;a composite particle

$\phi$ ; Higgs field ➡ $\phi = \langle \phi \rangle + \tilde{\phi}$

Higgs particle (discovered @2012) with mass=125 GeV

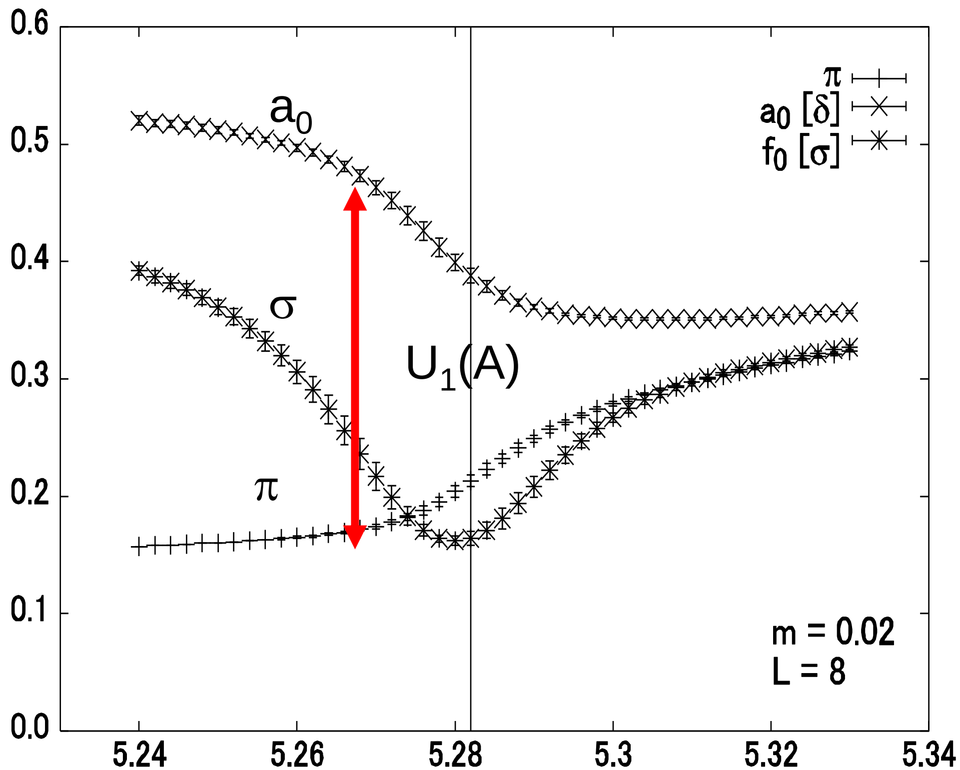
Is the Higgs a structureless elementary particle?

Cf. Lattice Calculation of the *generalized masses* :Screening masses

F. Karsch, Lect. Note Phys. **583** (2002), 209. $N_f = 2,\ 8^3 \times 4$; Staggered fermion

$$m_\sigma^2 = \chi_\sigma^{-1}$$

$$\chi_\sigma = \langle(\bar{q}q)^2\rangle$$



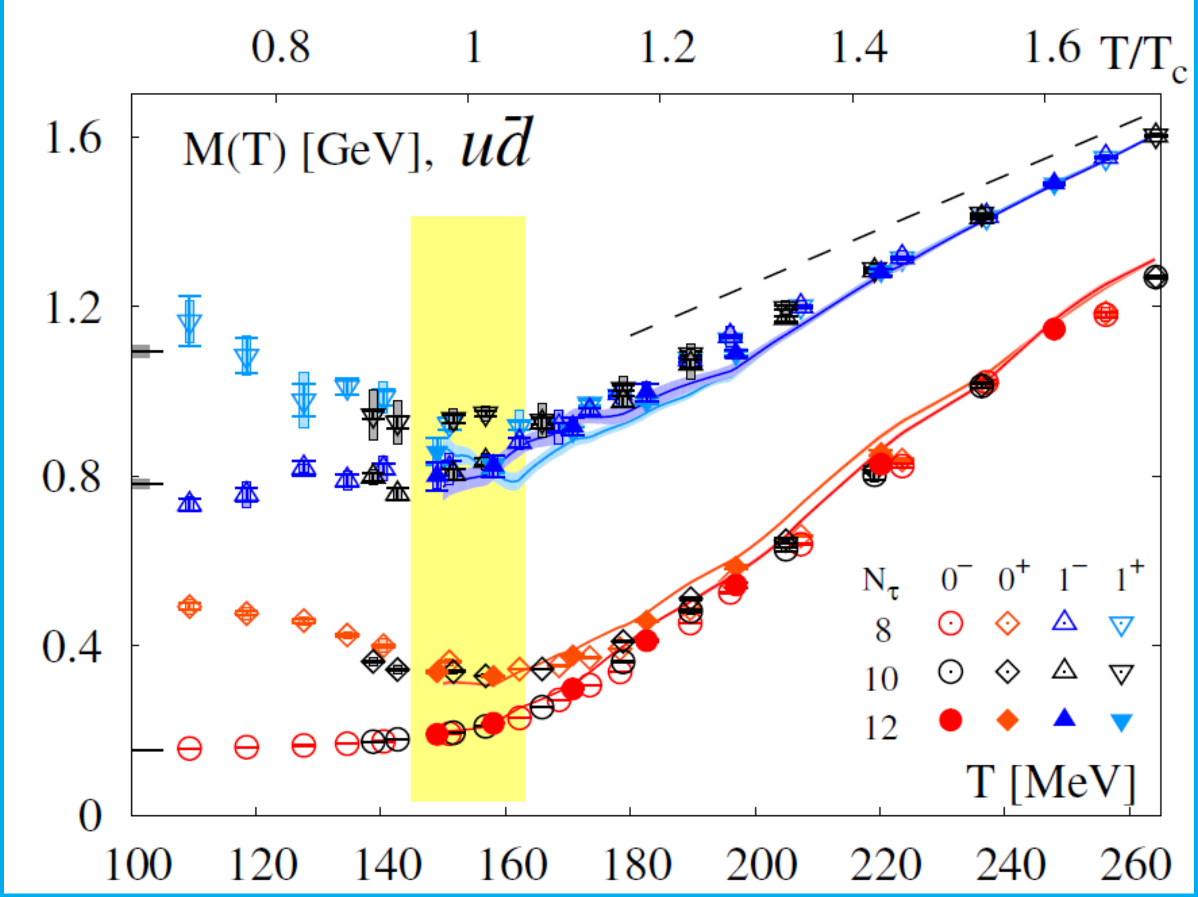

the softening of
the σ with increasing T

and

a degeneracy of the $\sigma$ and $\pi$ at high $T$

# Screening mass of light hadrons

Y.Maezawa, F.Karsch, S.Mukherjee and P.Petreczky,
**PoS LATTICE {¥bf 2015} (2016) 199**.




Positive parity and negative parity states tend to come close to each other.

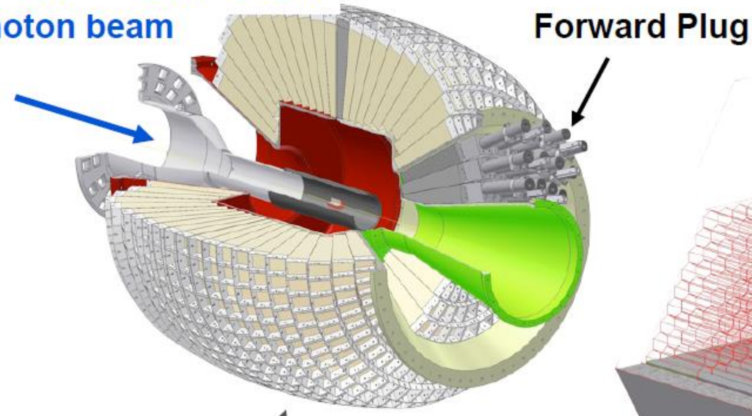
However, another simulation suggests different sceneario of chiral restoration as seen in the vector-axial channels; SCALAR collaboration, in preparation.

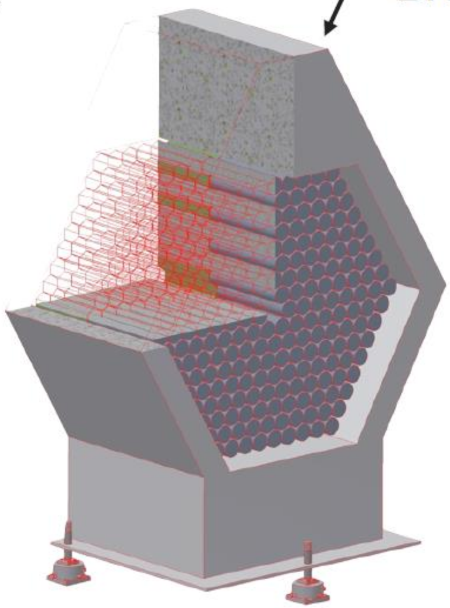
# Spectral change in vector-axial vector channels in association with partial restoration of chiral symmetry in hot and dense matter

# CBELSA/TAPS experiment

V. Metag
@MIN2016

$E_\gamma$=0.7 - 3.1 GeV
photon beam



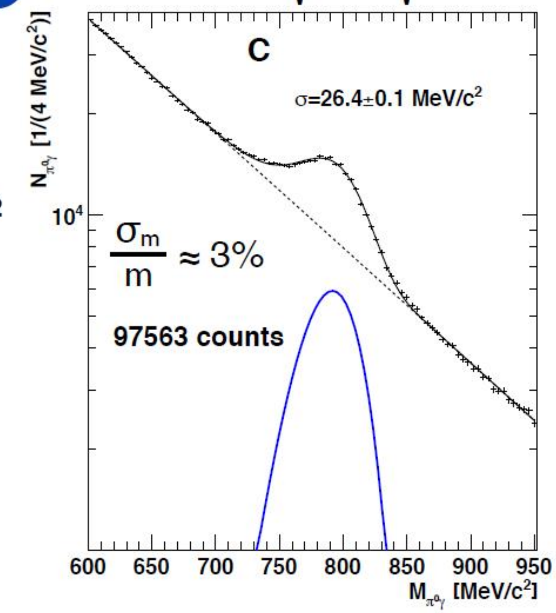

solid target: $^{12}$C and $^{93}$Nb

$4\pi$ photon detector: ideally suited for identification of multi-photon final states

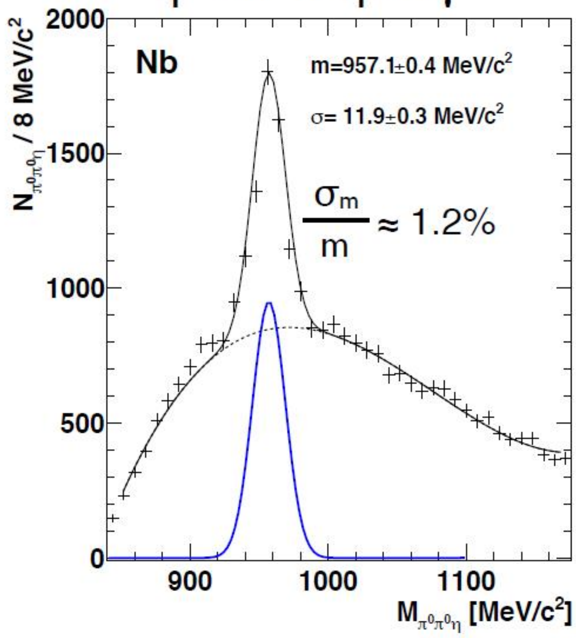
$\omega \rightarrow \pi^0\gamma \rightarrow 3\gamma$ BR 8.2%

$\eta' \rightarrow \pi^0\pi^0\eta \rightarrow 6\gamma$ BR 8.5%

$\omega \rightarrow \pi^0\gamma \rightarrow 3\gamma$

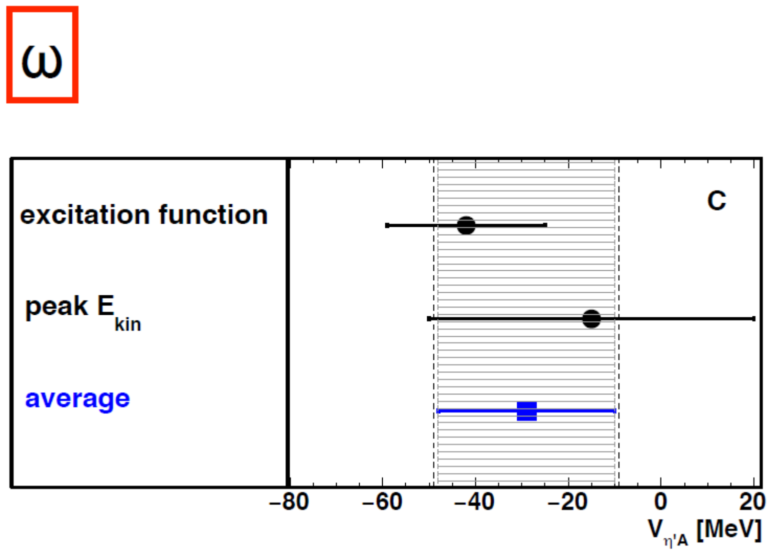



$\eta' \rightarrow \pi^0\pi^0\eta \rightarrow 6\gamma$

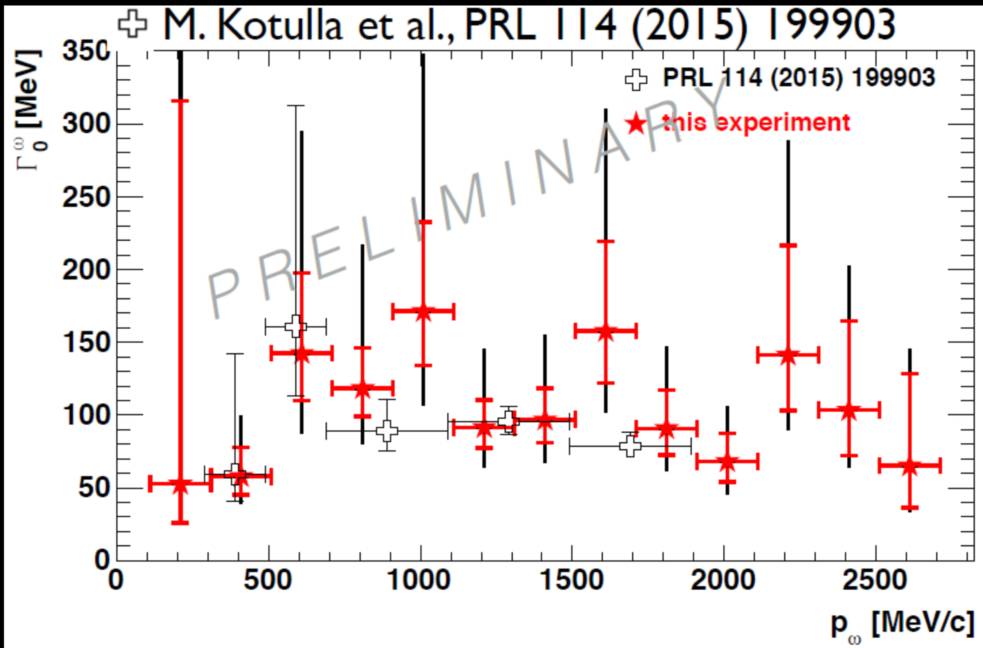


The real and imaginary part of the ω optical potential,
To be supposed to correspond to the mass shift and width.

Adapted from
V. Metag@MIN2016




$V_{\omega A}(\rho=\rho_0)$ =
−(29±19(stat)±20(syst)) MeV




ω in-medium width ≈ 50-150 MeV

What meson can be the chiral partner of the ω,
which is an axial vector composed (dominantly) of the u, d quarks?
Such mesons are denoted f_1.

Cf. The chiral partner of the ρ is a_1.

A summary of
mass and width of the (possible) chiral partner in the vector-axial vector channel

| $J^{PC}=1^{--}$ | mass | width | $J^{PC}=1^{++}$ | mass | width |
|---|---|---|---|---|---|
| $\rho$ | 770 | 150 | $a_1$ | 1260 | 250-600 |
| $\omega$ | 782 | 8.49 | $f_1$ | 1285 | 24.2 |
| $\phi$ | 1020 | 4.266 | $f_1$ | 1420 | 54.9 |

Mixing angle

$$|f_1(1285)\rangle = \cos\phi|n\bar{n}\rangle - \sin\phi|s\bar{s}\rangle,$$
$$|f_1^*\rangle = \sin\phi|n\bar{n}\rangle + \cos\phi|s\bar{s}\rangle,$$

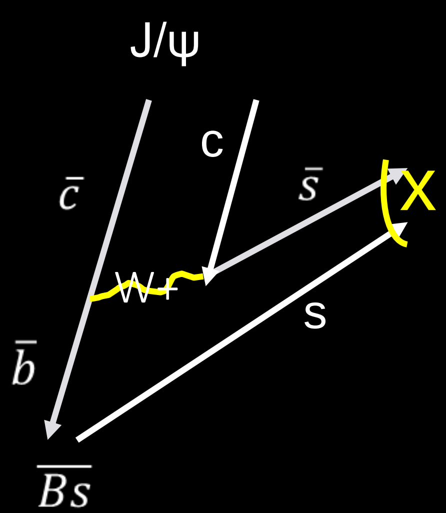
$$|n\bar{n}\rangle \equiv \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle)$$

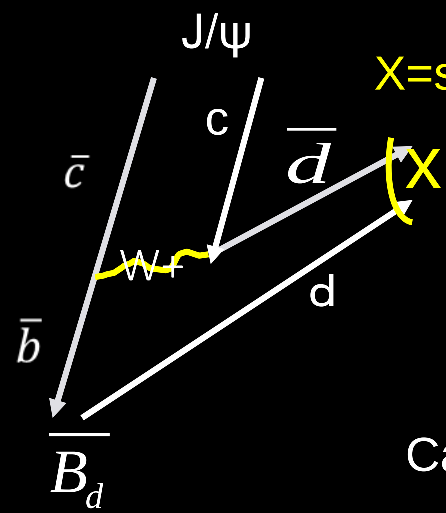
LHCb collab, PRL 112 (2014)

**Observation of $\bar{B}^0_{(s)} \to J/\psi f_1(1285)$ Decays and Measurement of the $f_1(1285)$ Mixing Angle**

$\Phi = \pm(24.0^{+3.1+0.6}_{-2.6-0.8})^\circ$ → f1(1285) is dominated by normal quarks!

Consistent and confirmed with the analysis by others: eg. F. Close and A.Kirk, Z. Phys. C76 (1997) for WA102 data, PRD 91 (2015) for the same LHCb data.




X=scalar or axial vector meson

Cabibbo suppressed

These processed are good for quantifying the quark content of X!

Close and Kirk, PRD (2015)

# f_1(1285) in the nuclear medium in QCD sum rules

**P. Gubler,S.-H. Lee, TK, PLB767 (2017), 336.**

$$\Pi_{\mu\nu}(q) = i\int d^4x e^{iqx}\langle T[j_\mu(x)j_\nu(0)]\rangle_\rho$$

$$J^q_\mu = \eta_{\mu\nu}\frac{1}{\sqrt{2}}\langle \bar{u}\gamma_\nu\gamma_5 u + \bar{d}\gamma_\nu\gamma_5 d\rangle \qquad J^s_\mu = \eta_{\mu\nu}\langle \bar{s}\gamma_\nu\gamma_5 s\rangle \qquad \eta_{\mu\nu} = q_\mu q_\nu/q^2 - g_{\mu\nu}$$

Twist-4 operators are neglected. The spectral function is represented by a delta function and a step function starting from a continuum threshold s_0.

$$\frac{m^2_{f_1}}{M^2} = \left[2\left(1+\frac{\alpha_s}{\pi}\right)E_2(s_0/M^2) - \frac{a}{M^2}E_1(s_0/M^2) + \frac{2(e-f)}{M^6}\right] \times \left[\left(1+\frac{\alpha_s}{\pi}\right)E_1(s_0/M^2) - \frac{a}{M^2}E_0(s_0/M^2) - \frac{-b+c+d}{M^4} - \frac{2(e-f)}{M^6}\right]^{-1}.$$

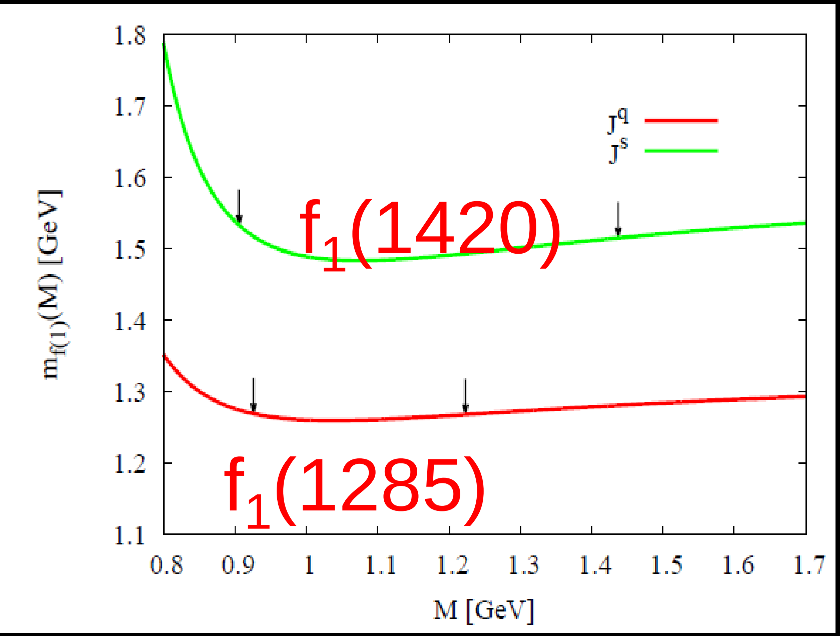
$$E_0(s_0/M^2) = 1 - e^{-s_0/M^2} \qquad E_1(s_0/M^2) = 1 - (1+\frac{s_0}{M^2})e^{-s_0/M^2}$$

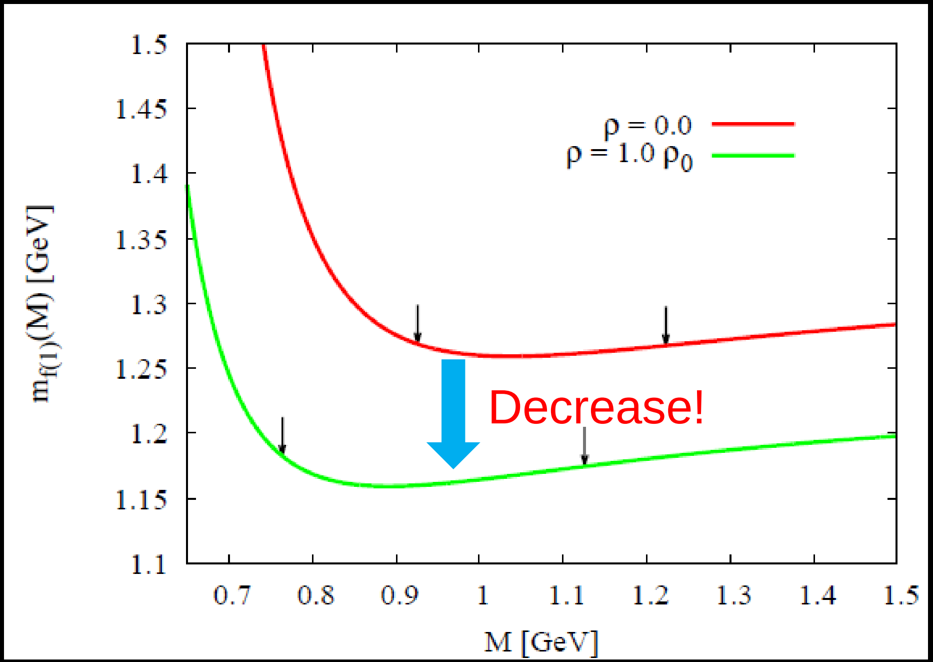
$$E_2(s_0/M^2) = 1 - (1+\frac{s_0}{M^2}+\frac{s_0^2}{2M^4})e^{-s_0/M^2}$$

| | |
|---|---|
| $\langle\bar{q}q\rangle_0$ | $(-0.248\ \mathrm{GeV})^3$ [26] |
| $\langle\bar{s}s\rangle_0$ | $0.8\times\langle\bar{q}q\rangle_0$ [27] |
| $m_q$ | 4.7 MeV [28] |
| $m_s$ | 95 MeV [28] |
| $\langle\frac{\alpha_s}{\pi}G^2\rangle_0$ | $0.012\ \mathrm{GeV}^4$ [29] |
| $M_N$ | 939 MeV |
| $\sigma_{\pi N}$ | $45\pm15$ MeV [24, 25, 30] |
| $\sigma_{sN}$ | 35 MeV [31] |
| $A^q_2$ | 0.62 [32] |
| $A^q_4$ | 0.066 [32] |

In the vacuum

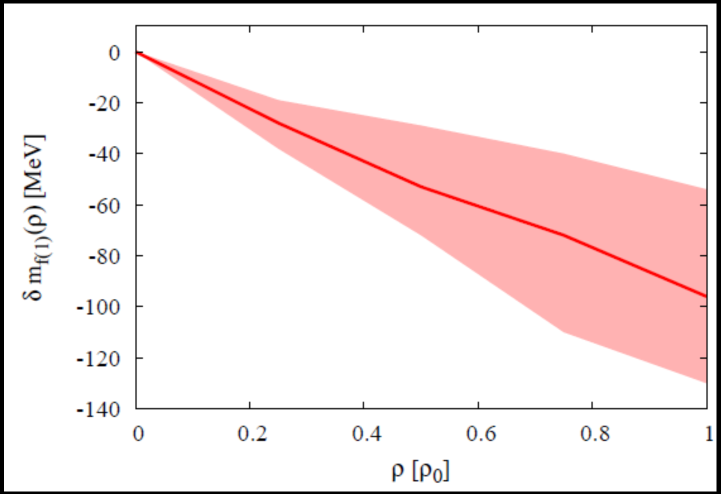
## The density dep. of f_1 mass




P. Gubler, S.-H. Lee, TK, PLB.767 (2017)

The uncertainty of the f_1 mass shift due to that of the π-N sigma term



# Conclusions in this part

・f_1(1285) may be identified as the chiral partner of ω:
f_1(1285) and f_1(1420) are almost ideally mixed.

・QCD sum rules show that f_1(1285) decreases its mass at finite density with an amount of tens of MeV.

・Viewing that the experimental result @CLAS suggests that ω shows a small mass shift of -29 MeV and a larger increase in the width of 70 MeV,

we arrive at the important conclusion that
partial restoration of chiral symmetry in nuclear medium could manifest itself
as a larger mass decrease of f_1(1285) and
a smaller decrease for ω
with an increasing width for both hadrons.

Experiment to produce f_1 in nuclei is considered.

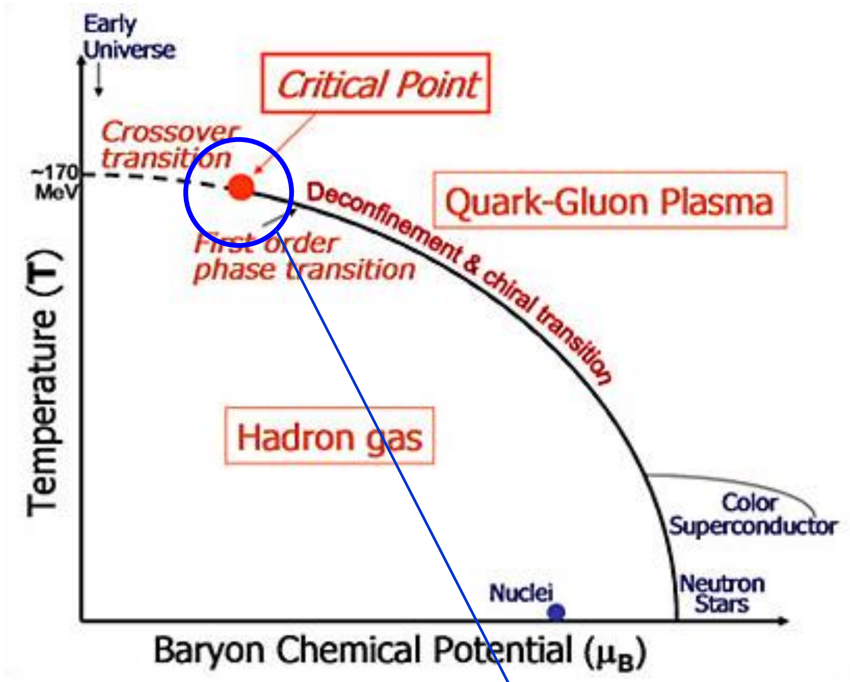
# Conclusions re $f_1(1285)$:

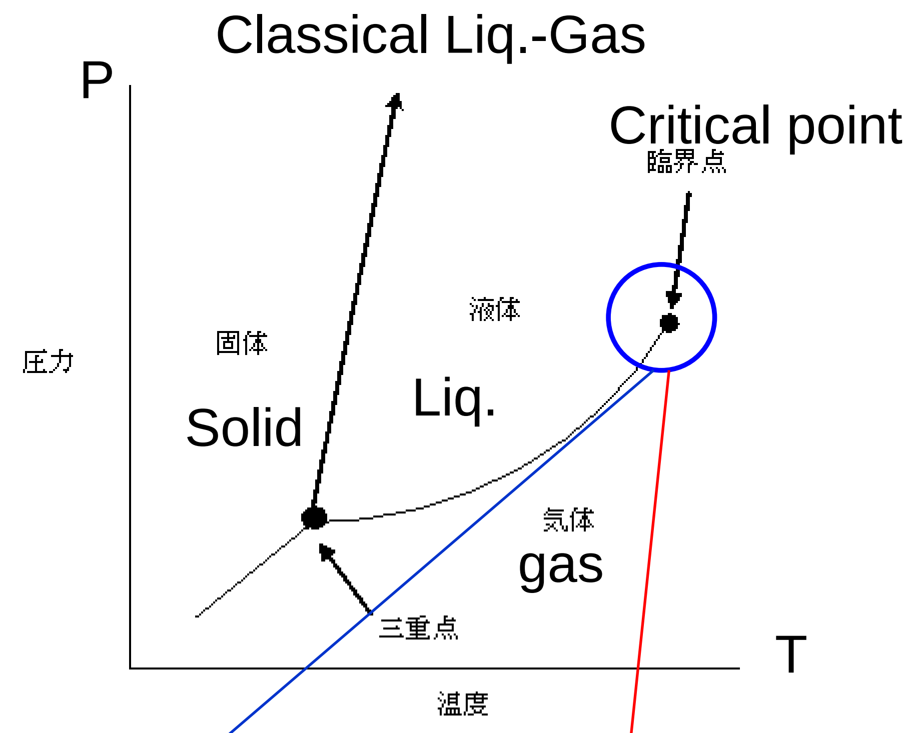
- The photoproduced meson CLAS sees at 1281 MeV is the $f_1(1285)$, not the $\eta(1295)$.
- It comes from the decay of $N^*$ or other non-$t$-channel processes.
- Can it be done in the nuclear medium?
- MIN theme of medium modifications: how is this axial-vector meson altered in the medium?

R. Schumacher@MIN2016

# The soft modes at the QCD critical point and possible precursor of inhomogeneous phase based on the Functional Renormalization Group method

Takeru Yokota, Kenji Morita and TK,
PTEP (2016) 073D01(arXiv:1603.02147);
and a paper in preparation




The same universality class; Z2

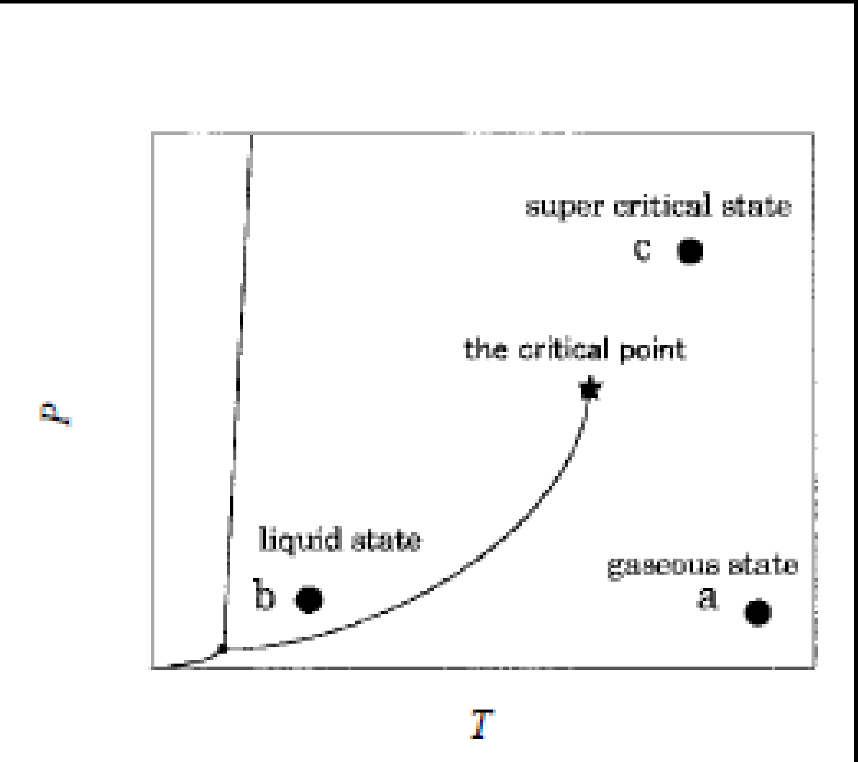
H. Fujii, PRD 67 (03) 094018;H. Fujii and M. Ohtani, Phys.Rev.D70(2004)
Dam. T. Son and M. A. Stephanov, PRD70 ('04) 056001

Density fluctuation is the soft mode of QCD critical point!

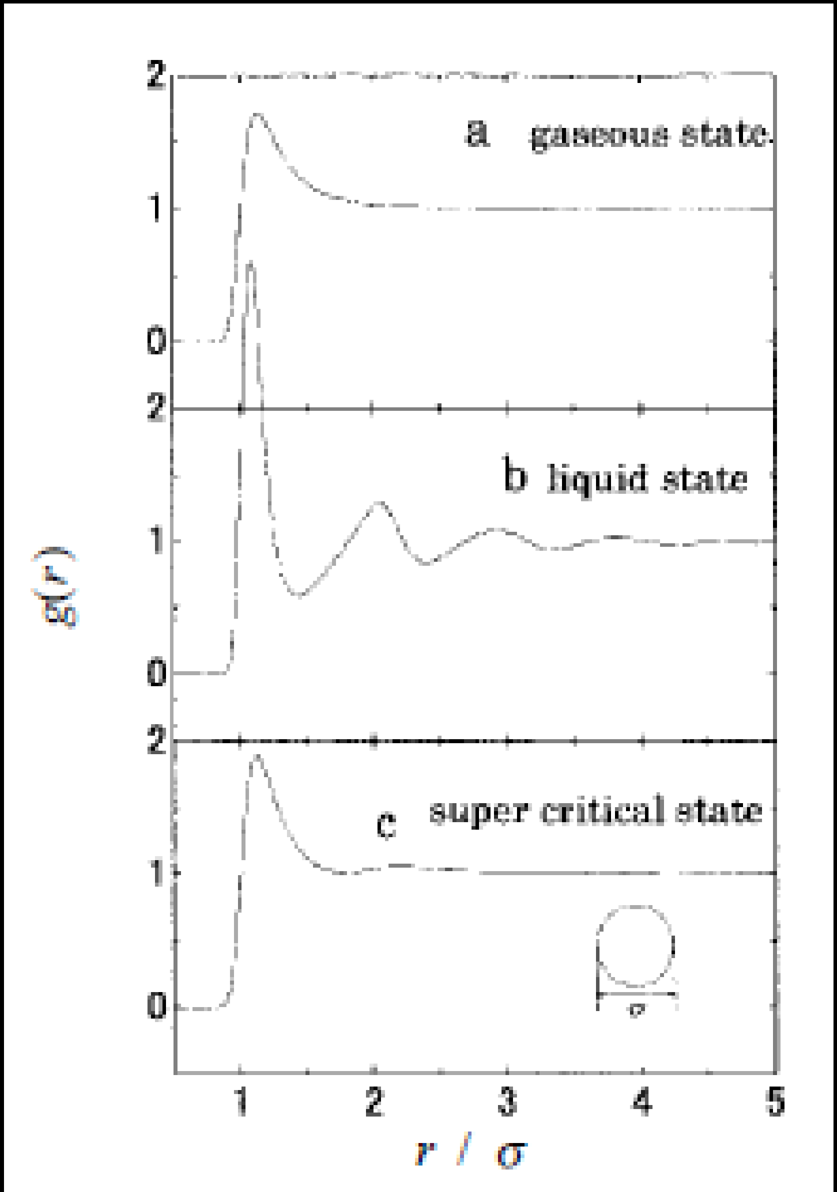
In classical liquid, a strong scattering of light;
→ critical opalescence.

# Density-density correlations in Liquid?

Ph. Diagram for Liq-Gas tr.




2-body correla. function g(r)




Masahar Ohba;
Netsu Sokutei 30(2), 65

# What is the soft mode at CP?

**Sigma meson has still a non-zero mass at CP.**
**This is because the chiral symmetry is explicitly broken.**

**What is the soft mode at CP?**

At finite density, scalar-vector mixing is present.

Spectral function of the chiral condensate
T-dependence ($\mu=\mu_{CP}$ )

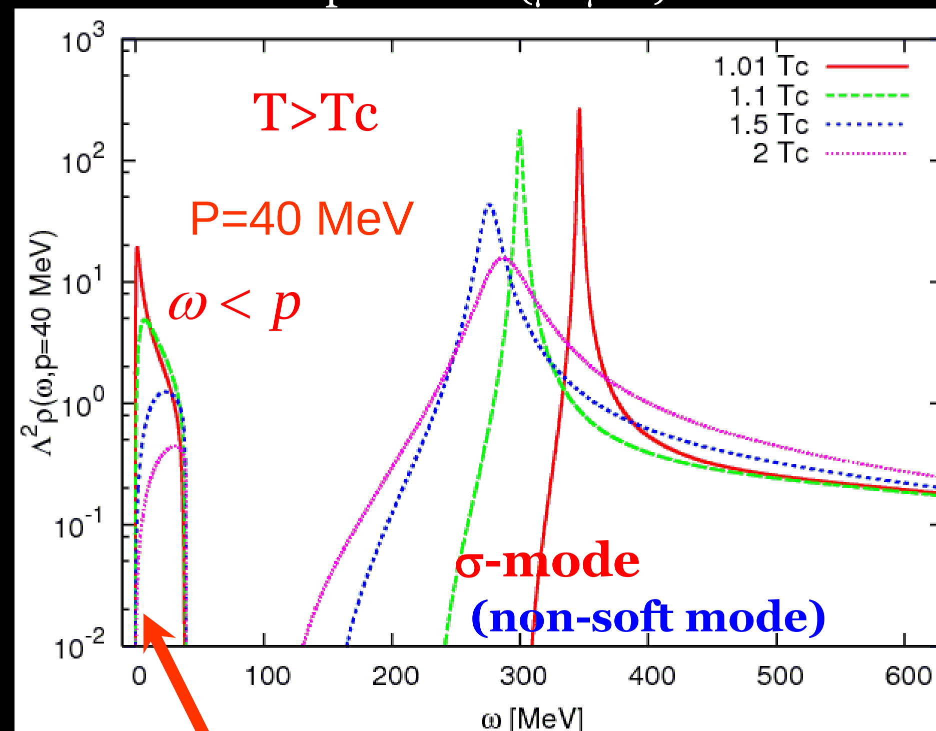
**Phonon mode in the space-like region softens at CP.**

**H. Fujii (2003)**
**H. Fujii and M.Ohtani(2004)**

See also, D. T. Son and M. Stephanov (2004)

does not affect particle creation in the time-like region.

**It couples to hydrodynamical modes,**
leading to interesting dynamical critical phenomena.




**Space-like region** $\omega < p$

**(the soft modes)**

# Functional RG approach with meson-quark model

Takeru Yokota, Kenji Morita and TK,
PTEP (2016) 073D01(arXiv:1603.02147)

$$S_\Lambda\left[\overline{\psi},\psi,\phi=(\sigma,\vec{\pi})\right]=\int_0^{\frac{1}{T}}d\tau\int d^3\vec{x}\left\{\overline{\psi}\left(\partial\!\!\!/+g_s(\sigma+i\vec{\tau}\cdot\vec{\pi}\gamma_5)-\mu\gamma_0\right)\psi+\frac{1}{2}(\partial_\mu\phi)^2+V_\Lambda(\phi^2)-c\sigma\right\}$$

$$V_\Lambda(\phi^2)=\frac{1}{2}m_\Lambda^2\phi^2+\frac{1}{4!}\lambda(\phi^2)^2$$

| $\Lambda$ | $m_\Lambda/\Lambda$ | $\lambda_\Lambda$ | $c/\Lambda^3$ | $g_s$ |
|---|---|---|---|---|
| 1000MeV | 0.794 | 2.00 | 0.00175 | 3.2 |

R. Tripolt, L. Smekal, J. Wambach, PRD90 (2014)

Vacuum value

| $M_q$ | $M_\pi$ | $M_\sigma$ | $\sigma_0$ |
|---|---|---|---|
| 286MeV | 137MeV | 496MeV | 93MeV |

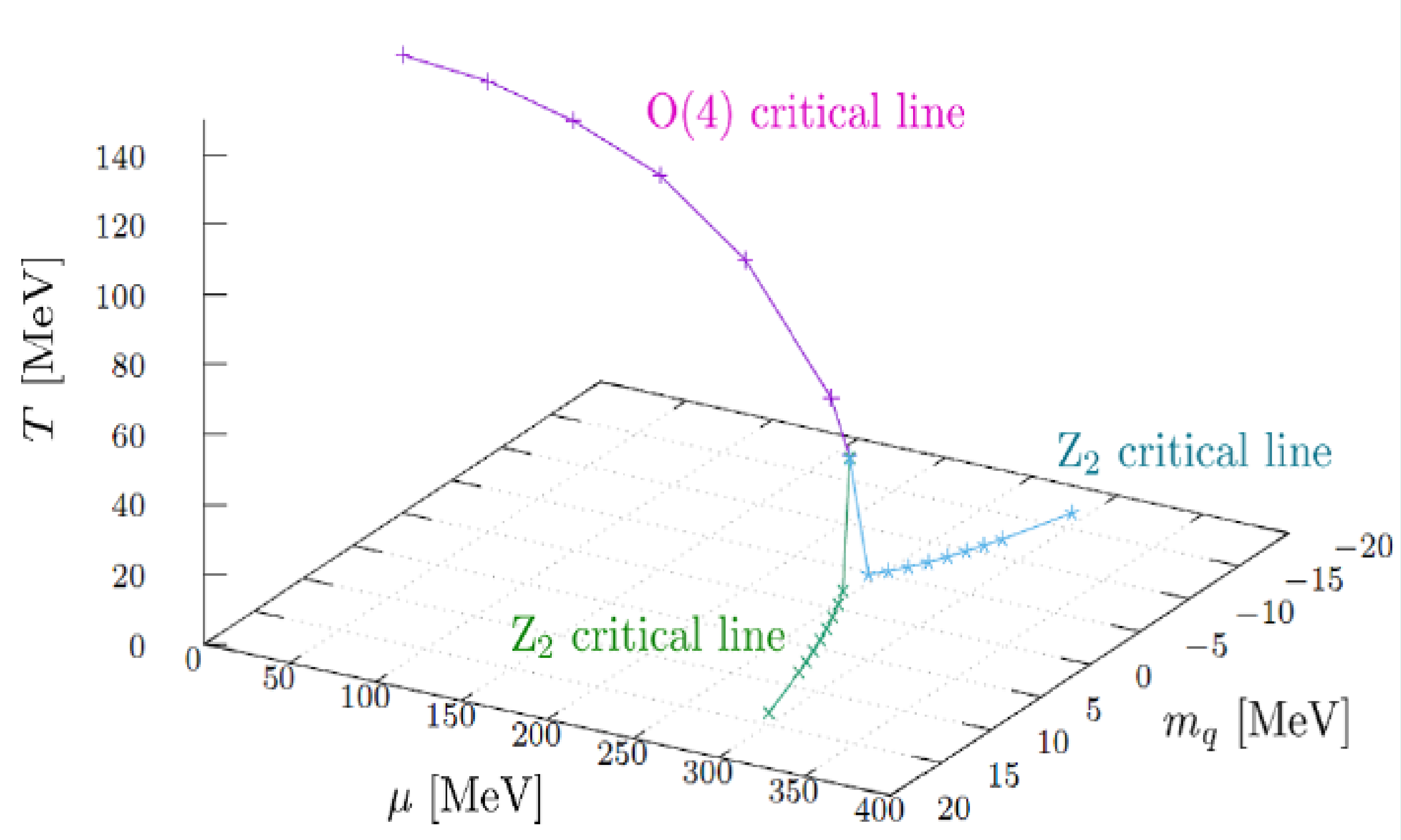
$M_q$ is the constituent quark mass.

$$M_q=g_s\sigma_0$$

$M_\pi$ and $M_\sigma$ are the screening masses.

$$M_\pi=\frac{1}{\sigma_0}\frac{\partial U_0}{\partial\sigma}(\sigma_0)\quad M_\sigma=\frac{\partial^2 U_0}{\partial\sigma^2}(\sigma_0)$$

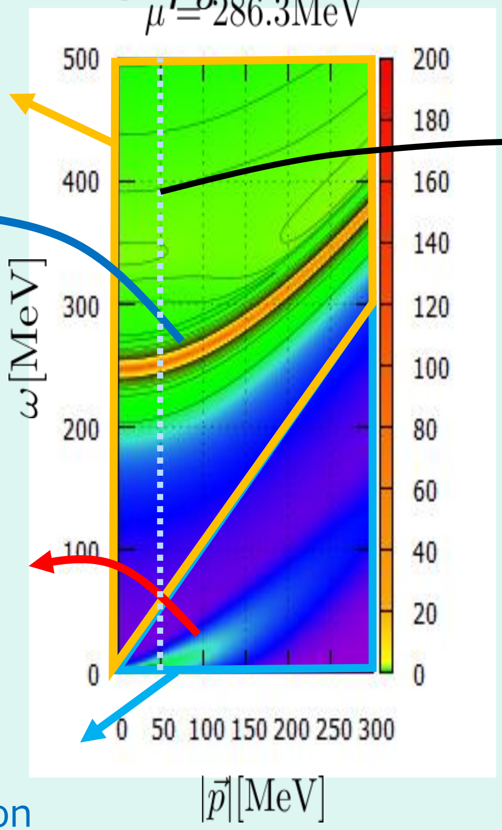
# Quark-mass dependence of the phase transition



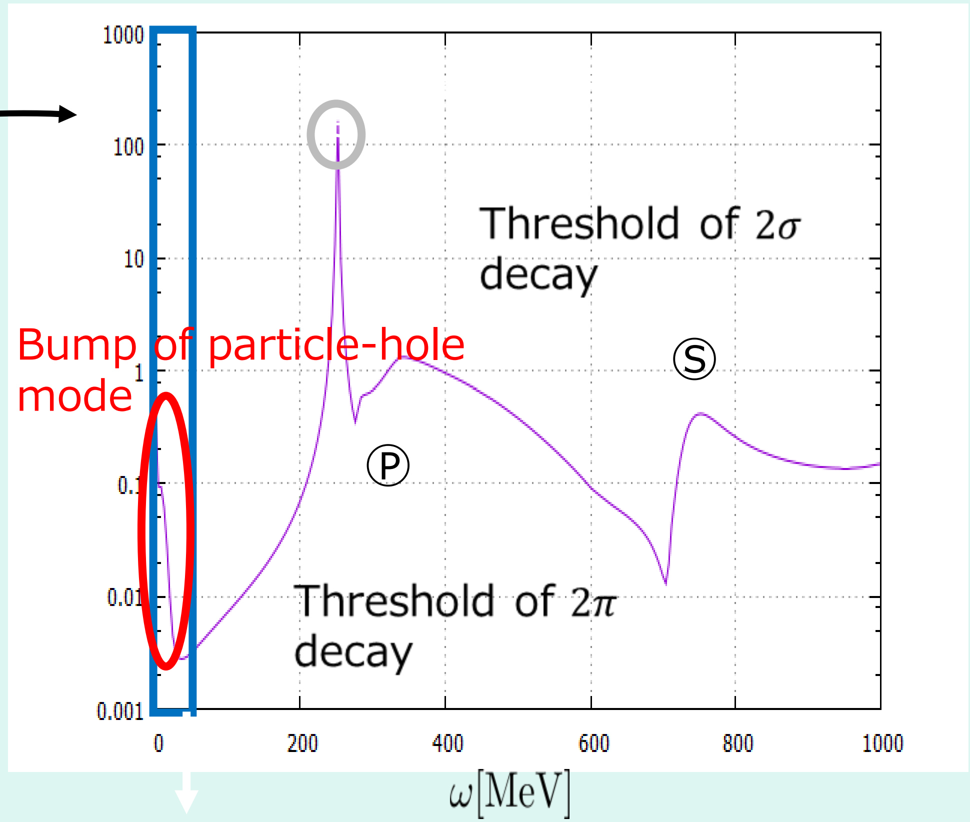
# An example of the sigma channel spectral function $\rho_\sigma$ near

- We fix $T = 5.1$ MeV below. (At this temperature, the transition chemical potential is $\mu_t = 286.69$ MeV)

**Contour map of $\rho_\sigma$**




**$\rho_\sigma$ when $|\vec{p}| = 50$ MeV**

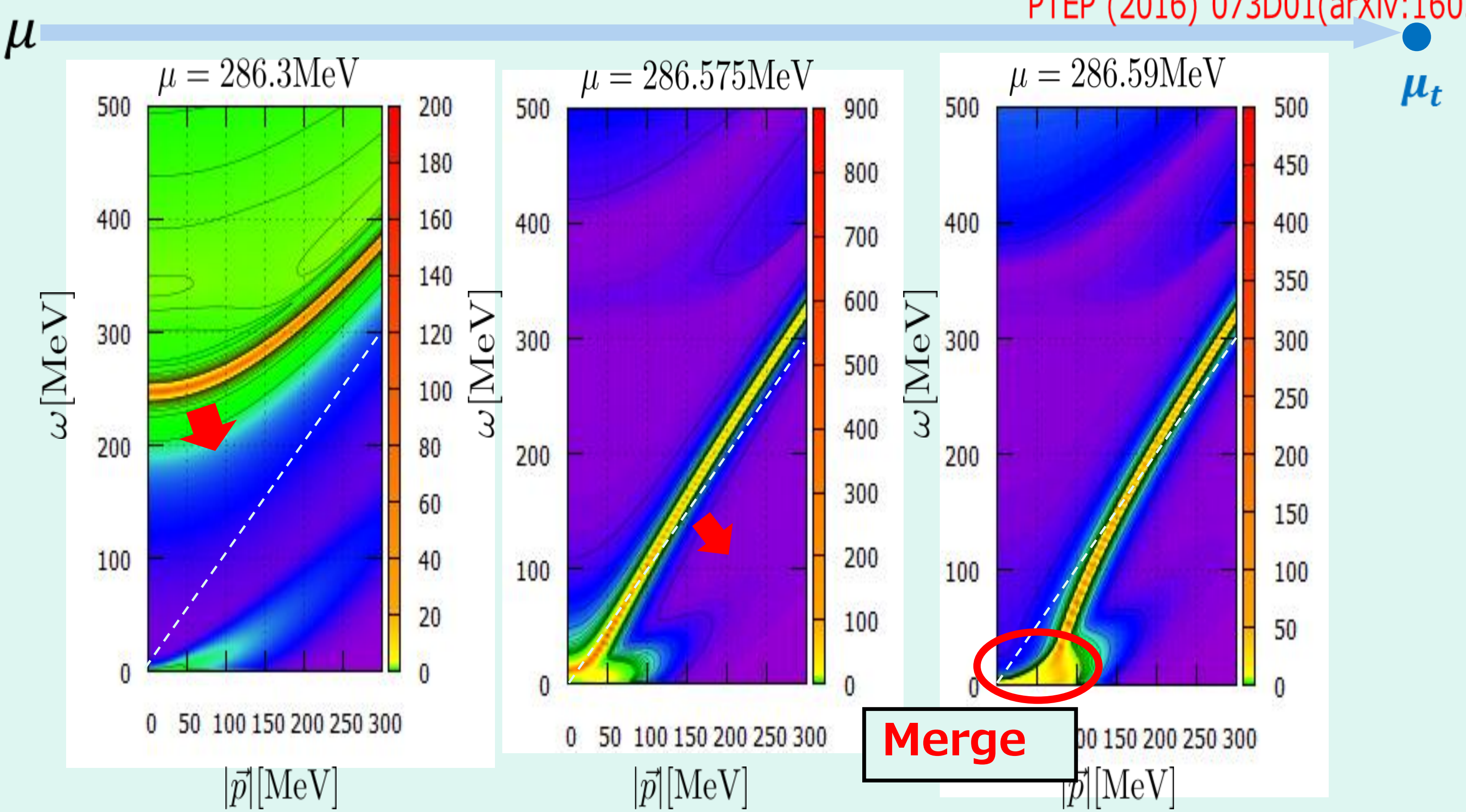



Takeru Yokota, Kenji Morita and TK, PTEP (2016) 073D01(arXiv:1603.02147)

# Anomalous softening of the sigma once located in the time-like region to merge into the phonon mode

- We fix $\boldsymbol{T} = \mathbf{5.1}$ MeV. (At this temperature, the transition chemical potential is $\boldsymbol{\mu_t} = \mathbf{286.69}$ MeV)

Takeru Yokota, Kenji Morita and TK,
PTEP (2016) 073D01(arXiv:1603.02147)




- As the system approaches the QCD CP, the dispersion relation of sigma mesonic mode shifts to low-energy region and merges with the bump of the particle-hole modes in the space-like momentum region.

The merge of the sigma and phonon and appearance of the tachyonic mode @ m_q=8.88 MeV.
However, the simulation with smaller current quark mass shows no such anomalous behavior: The soft mode is the phonon mode with a support in the space-like region, while the sigma-mesonic mode remains in the time-like region.

Appearance of an acausal tachyonic mode at finite momenta!

↓

Suggesting an instability of the system, to an inhogeneous phase,with modurated sigma field as well as the density, no pion field, i.e., of the real kink type.

**T. Yokota, K. Morita and T.K., in preparation**

c.f. Real Kink type: D.Nickel, M. Buballa (2010).
DCDW: Nakano, Tatsumi (2005); chiral limit.

# Brief summary and concluding remarks

1. Restoration of chiral symmetry means the degeneracy of the correlation functions and hence the spectral functions of the chiral partner; i.e., scalar-pseudo-scalar or vector-axial-vector.

2. U(1)_A anomaly may be effectively (partially) restored in hot and/or dense matter in the spectra of hadronic modes, such as pion and a_0.

3. f_1(1285) may be identified as the chiral partner of ω, and QCD sum rules show that f_1(1285) decreases its mass at finite density with an amount of tens of MeV.

4. Viewing that the experimental result @CLAS suggests that ω shows a small mass shift of -29 MeV and a larger increase in the width of 70 MeV, a possible scenario of partial restoration of chiral symmetry in nuclear medium is a larger mass decrease of f_1(1285) and a smaller decrease for ω with an increasing width for both hadrons.

5. The FRG analysis of the spectral function and the dispersion relations of the **modes shows that the soft mode of QCD CP** with realistic current quark mass is the density fluctuations, i.e., phonons with the spectral support in the space-like region.

6. As for the sigma mesonic mode that is the soft mode in the chiral limit, our FRG calculation shows that the dispersion relation of the sigma mesonic mode with a support in the time-like region shows a complicated temperature dependence for relatively small current quark mass, including anomalous softening diving into the space-like region to merge into the phonon mode.

7. It eventually exhibits a tachyonic nature at finite momenta for the physical quark mass, which strongly suggests the instability of the system and a phase transition to an inhomogeneous phase (so called the real kink phase) already at smaller quark mass.

8. No such behavior is seen in the pion channel.

BACK UPS

PDG2016

Small width

Axial vector

# $f_1(1285)$

$I^G(J^{PC}) = 0^+(1^{++})$

Mass $m = 1282.0 \pm 0.5$ MeV (S = 1.8)

Full width $\Gamma = 24.1 \pm 1.0$ MeV (S = 1.3)

| $f_1(1285)$ DECAY MODES | Fraction ($\Gamma_i/\Gamma$) | Scale factor/ Confidence level | $p$ (MeV/$c$) |
|---|---|---|---|
| $4\pi$ | $(33.1^{+2.1}_{-1.8})$ % | S=1.3 | 568 |
| $\pi^0\pi^0\pi^+\pi^-$ | $(22.0^{+1.4}_{-1.2})$ % | S=1.3 | 566 |
| $2\pi^+2\pi^-$ | $(11.0^{+0.7}_{-0.6})$ % | S=1.3 | 563 |
| $\rho^0\pi^+\pi^-$ | $(11.0^{+0.7}_{-0.6})$ % | S=1.3 | 336 |
| $\rho^0\rho^0$ | seen | | † |
| $4\pi^0$ | $< 7 \times 10^{-4}$ | CL=90% | 568 |
| $\eta\pi^+\pi^-$ | $(35 \pm 15)$ % | | 479 |
| $\eta\pi\pi$ | $(52.4^{+1.9}_{-2.2})$ % | S=1.2 | 482 |
| $a_0(980)\pi$ [ignoring $a_0(980) \to K\overline{K}$] | $(36 \pm 7)$ % | | 238 |
| $\eta\pi\pi$ [excluding $a_0(980)\pi$] | $(16 \pm 7)$ % | | 482 |
| $K\overline{K}\pi$ | $(9.0 \pm 0.4)$ % | S=1.1 | 308 |
| $K\overline{K}^*(892)$ | not seen | | † |
| $\pi^+\pi^-\pi^0$ | $(3.0 \pm 0.9) \times 10^{-3}$ | | 603 |

Small amount of strangenesee content

Possible chiral partner of Φ(1020)?

**$f_1(1420)$** [p]

$I^G(J^{PC}) = 0^+(1^{++})$

Mass $m = 1426.4 \pm 0.9$ MeV (S = 1.1)
Full width $\Gamma = 54.9 \pm 2.6$ MeV

| **$f_1(1420)$ DECAY MODES** | Fraction ($\Gamma_i/\Gamma$) | $p$ (MeV/$c$) |
|---|---|---|
| $K\overline{K}\pi$ | dominant | 438 |
| $K\overline{K}^*(892)+$ c.c. | dominant | 163 |
| $\eta\pi\pi$ | possibly seen | 573 |
| $\phi\gamma$ | seen | 349 |

# Nonperturbative properties of QCD

Gell-Mann-Oakes-Renner

$$f_\pi^2 m_{\pi^\pm}^2 \simeq -\hat{m}\langle \bar{u}u + \bar{d}d \rangle$$

$$\hat{m}=(m_u+m_d)/2$$

using

$f_\pi$=93 MeV and $\hat{m}$(1GeV)=(7 ± 2) MeV,

We have

$$\langle \bar{u}u \rangle \simeq \langle \bar{d}d \rangle \simeq [-(225 \pm 25) MeV]^3 \quad \text{at} \quad \mu^2 = 1\text{GeV}$$

Chiral symmetry is spontaneously broken in QCD vacuum.

QCD sum rules for heavy-quark systems,

$$\langle \frac{\alpha_s}{\pi} F_{\mu\nu}^a F_a^{\mu\nu} \rangle = (350 \pm 30 \text{ MeV})^4$$

Cf. Heavy scalar mesons and glue balls

$$f_0(1370) = +0.36|G\rangle - 0.51|s\bar{s}\rangle - 0.78|n\bar{n}\rangle$$

$$f_0(1500) = -0.03|G\rangle + 0.84|s\bar{s}\rangle - 0.54|n\bar{n}\rangle$$

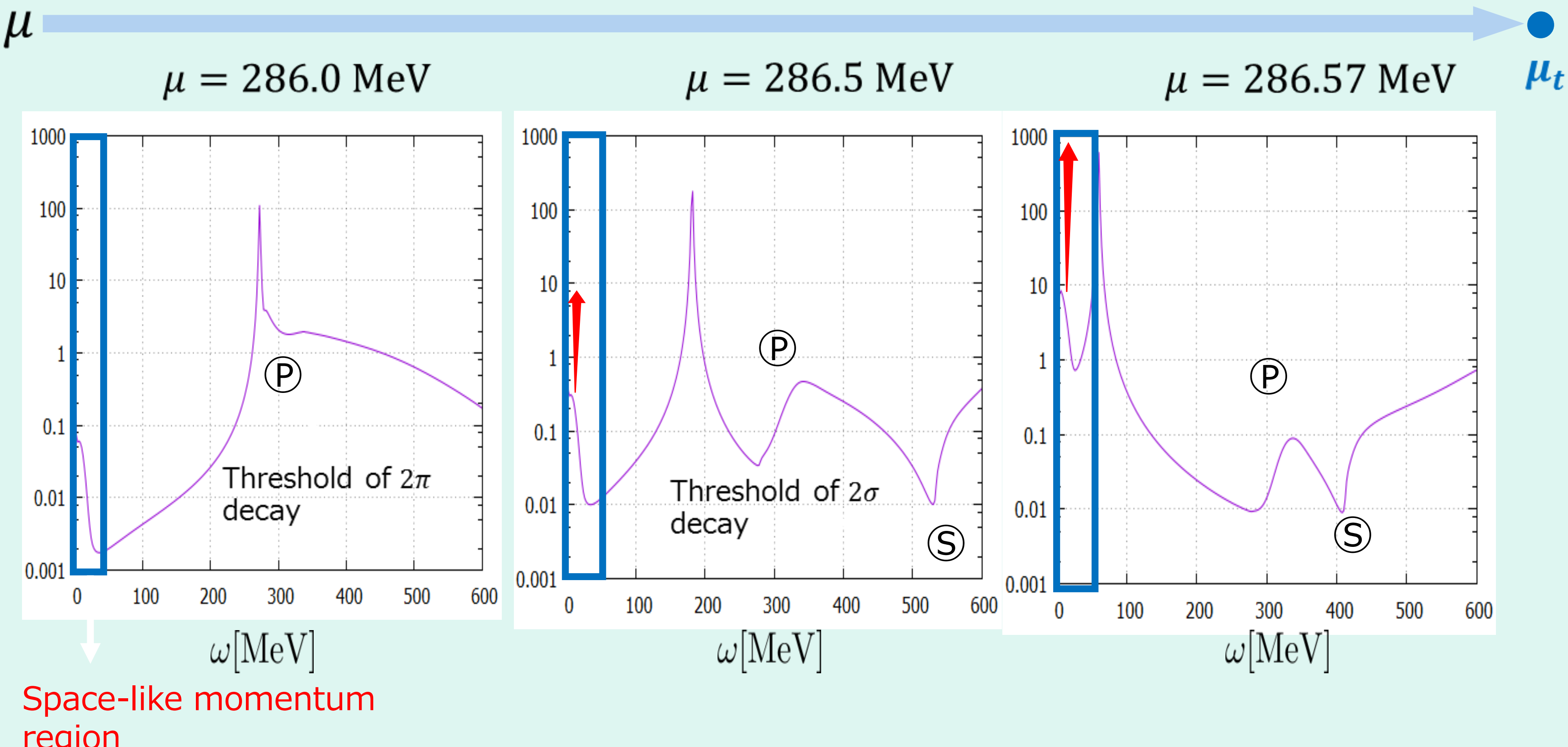
$$f_0(1710) = +0.93|G\rangle + 0.18|s\bar{s}\rangle + 0.32|n\bar{n}\rangle$$

Consistent with a recent lattice cal.:

H.-Y.Cheng et al, PRD 92, 094006 (2015)

# Development of the phonon mode near the QCD CP

- We fix $\boldsymbol{T} = \mathbf{5.1}$ MeV and $|\vec{\boldsymbol{p}}| = \mathbf{50}$ MeV. (At this temperature, the transition chemical potential is $\boldsymbol{\mu_t} = \mathbf{286.69}$ MeV)




- The particle-hole mode becomes soft.

Takeru Yokota, Kenji Morita and TK, PTEP (2016) 073D01(arXiv:1603.02147)

# Two-body gluon correlation function on the lattice

Harvey B. Meyer, **Phys.Rev.D79,011502 (2009);**

**See also** N.Iqbal and H.B.Meyer, JHEP {¥bf 0911} (2009) 029